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MODEL FOR DROPWISE CONDENSATION ON RANDOMLY DISTRIBUTED SITES

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Abstract—A model for heat transfer during dropwise condensation on randomly distributed nucleation sites was developed and simulated on a digital computer. The theory accounted for drop nucleation and growth, coalescence with neighbors, removal, and renucleation on sites exposed by the removal and coalescence mechanisms. For steady state drop condensation, the theory showed that small drops grow by vapor condensation, and that larger drops grow predominently by coalescences. Larger heat-transfer coefficients than have been observed to date should be possible if condenser surfaces containing between 10^7–10^8 sites/cm^2 and drop removal techniques for very small drops ($< 5 \times 10^{-4}$ cm) can be developed.

INTRODUCTION

TO EXPLAIN the high heat-transfer coefficients that are observed during dropwise condensation, Jakob [1] suggested that the vapor molecules first condense to form a discontinuous film which very quickly varies in thickness and rolls itself together to form drops. This concept was later used by Baer and McKelvey [2] and Welch and Westwater [3] who suggested that the film between drops fractures at a critical thickness, and that surface-tension forces then quickly aid formation of small drops. This mechanism suggested that the critical film thickness was the controlling variable for heat transfer, and the heat is predominately transferred between drops which subsequently grow due to numerous coalescences. From heat-transfer data, the critical film thickness is estimated to be several microns for a typical case, which suggests that a layer of this thickness could be observed experimentally. The recent experimental observations of Umur and Griffith [4] conclude that no film greater than a monolayer in thickness exists between drops. In addition, both Eucken [5] and Emmons [6] have proposed slightly different experimentally unverified mechanisms for heat transfer between drops.

McCormick and Baer [7, 8] have recently proposed that heat is transferred through numerous very small drops, many submicroscopic, which are randomly nucleated on active sites on the condenser surface. Nucleation sites are continually being exposed by numerous drop coalescences and by large drops detaching or sliding from the surface. Experimental evidence in support of this mechanism has been accumulated and can be found in references [4, 7, 8]. To date, no satisfactory heat-transfer model for dropwise condensation has been proposed. Attempts by Fatica and Katz [9] and later by Sugawara and Michiyoshi [10] have produced semiquantitative theories which unfortunately are extremely difficult to check by experiment.

CONDENSATION MODEL

In this model, condensation occurs only onto drops which form on active sites. It is assumed that the rate-limiting mechanism is heat conduction through the drop, so that the rate of condensation onto a given drop is proportional to the exposed area of the drop, and inversely proportional to the length of the heat-transfer path from a point on the exposed surface of

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the drop to a point on the surface which supports the drop. If the shape of the drop is independent of its size, the condensation rate, or rate of change of drop volume V, will be proportional to some characteristic length r in the drop;

$$\frac{dV}{dt} = a_1 r. \tag{1}$$

For any shape,

$$V = a_2 r^3. \tag{2}$$

Combining equations (1) and (2) gives

$$\frac{dr}{dt} = \frac{a_1}{3a_2}\frac{1}{r}. \tag{3}$$

If the initial condition is $r = 0$ at $t = 0$, the solution is

$$r = \left[\frac{2a_1}{3a_2}t\right]^{\frac{1}{2}}. \tag{4}$$

The value of the constants which depend on the shape of the drop have previously been calculated [9]. In this model, the drops were assumed to be hemispherical. For a hemisphere, if r is taken to be the radius, and it is assumed that both surfaces of the drop remain at constant temperature,

$$\frac{2a_1}{3a_2} = 4{\cdot}15\frac{k\Delta T}{H_c\rho} \tag{5}$$

where k is the liquid thermal conductivity, ΔT is the difference in temperature between the two surfaces of the drop, H_c is the heat of condensation and ρ is the liquid density.

Consider the case of condensation without coalescence on the underside of a horizontal surface. If a drop is not disturbed by coalescence with a neighboring drop, it will grow according to equation (4) until some time θ at which its size R is too great to be supported by surface forces. It will then fall and the process will be repeated. At the steady state, the ages of the various drops will be uniformly distributed between $t = 0$ and $t = \theta$. r^2 will also be uniformly distributed, and r will have the distribution shown in Fig. 1.

According to equation (1), the condensation rate for each drop is proportional to the characteristic length r. Therefore, the overall condensation rate per unit area, C, can be

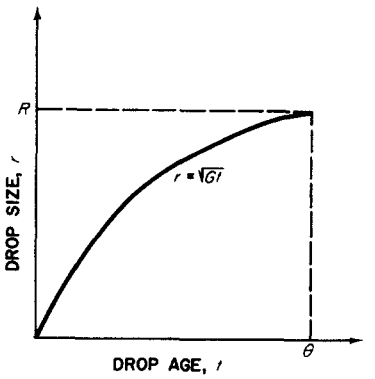

FIG. 1. Drop growth without coalescence.

calculated by multiplying the number of drops per unit area, N, by the condensation rate for a drop with the mean characteristic length $\bar{r}$.

$$C = a_1 N\bar{r}. \tag{6}$$

The mean characteristic length will depend on the state of crowding on the plate. If the population is dense, most drops will coalesce before they reach the maximum size R. The state of crowding can be represented by a dimensionless parameter, β, which is proportional to the area covered by a drop of maximum size divided by the average area per site, N^{-1}

$$\beta = NR^2. \tag{7}$$

Limiting forms will be derived for small and large values of β. If either the number of sites per unit area, N, or the critical drop size, R, is small, coalescence will be very unlikely, and the value of β will be small. If coalescence does not occur, $\bar{r}$ may be found by invoking the ergodic hypothesis for the steady state and integrating the distribution of Fig. 1.

$$\bar{r} = \frac{1}{\theta}\int_0^\theta r\,dt = \frac{2R}{3} \quad \text{or} \quad \frac{\bar{r}}{R} = 0{\cdot}667, \beta \ll 1. \tag{8}$$

If the number of sites per unit area or the maximum drop size is large, each drop will probably cover several nucleation sites. The value of r will be zero for sites covered in this manner. This will have a net effect of decreasing $\bar{r}$. For example, doubling the characteristic length of a drop will cause it to cover four times as much area, and four times as many sites, and will therefore decrease the average radius by a factor of one-half, approximately. That is, $\bar{r} \propto R^{-1}$ for a constant value of N. Similarly, if additional nucleation sites are added to an already crowded surface, most of them will fall under drops which are already present. Because $\bar{r}$ is averaged over all the sites, the net effect will be to decrease $\bar{r}$. Thus, $\bar{r} \propto N^{-1}$ for a constant value of R. Combining these two arguments yields,

$$\frac{\bar{r}}{R} \propto \frac{1}{\beta} \beta \gg 1. \tag{9}$$

Substituting equations (8) and (9) into (6) makes possible the following conclusions: For low values of β, the condensation rate is proportional to both the site density, N, and the maximum drop size, R. For high values of β, the condensation rate is independent of the site density, and is inversely proportional to the maximum drop size. It also follows that, given a site density, N, there is some drop size, R, which maximizes the condensation rate. An analytical solution for the relationship between the expected value of $\bar{r}/R$ and β for randomly distributed sites would be quite difficult. However, this problem lends itself well to digital computer simulation by a Monte Carlo technique. The result of such a simulation is presented below.

COMPUTER SIMULATION

The condensation process described above was simulated on a digital computer. Two hundred nucleation sites were chosen randomly on a 100 × 100 grid. The result is shown in Fig. 2. A grid of this size allowed a wide range of β values to be used while the computational effort was kept within feasible limits. N was held constant and the critical drop size R ranged from 0·5 to 18·8 yielding values of β from 0·005 to 7·07. The drops grew according to equation (3), and were removed when the radius of a drop exceeded the critical radius R. When two drops touched during the growth process, they coalesced onto the site of the larger drop. A typical steady-state case illustrating both drop coalescence, drop removal, and new drop nucleation on vacated sites is shown in Fig. 3.

To obtain a discrete form of the growth law for digital computation, equation (4) was put into finite-difference form.

$$\Delta(r^2) = r_{\text{new}}^2 - r_{\text{old}}^2 = \frac{2a_1}{3a_2} \Delta t.$$

If a growth rate G is defined to be

$$G = \frac{2a_1 \Delta t}{3a_2} \tag{10}$$

then,

$$r_{\text{new}} = [(r_{\text{old}})^2 + G]^{\frac{1}{2}}. \tag{11}$$

G was chosen for each R so that the radius of the drop grew from zero to its critical size in approximately 25 iterations.

At the beginning of the program, a "Neighbor List" for each site "i" was formed. This list contained the names of all sites, the drops on which could coalesce with a drop at site i. The distance of each point j from point i was calculated according to,

$$D_{i,j} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{\frac{1}{2}}$$
$$i = 1, 2, \ldots 199 \tag{12}$$
$$j = i + 1, i + 2, \ldots 200.$$

If $D_{i,j}$ was less than $2R$, the jth site was included on site i's Neighbor List. When a site became covered by a drop centered at another site, it was entered on a "Covered Site List". Sites on this list were not allowed to grow drops until the drop covering them was removed. A flow diagram for the simulation is presented in

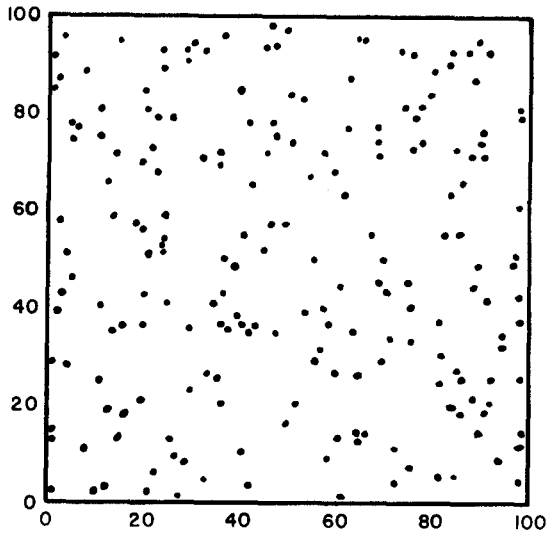

FIG. 2. Location of two hundred random sites for drop nucleation as generated by the computer.

Fig. 4. The program was written in ALGOL and was run on a UNIVAC-1107 digital computer. A detailed flow chart and the computer program are available from the American Documentation Institute.* The use of a finite grid size introduces some error because the points near the edges have an artificially low number of neighbors. An estimate of the edge effect on $\bar{r}$ was made by also using a 50 × 50 grid, on which the edge effect should be twice as large as on the 100 × 100 grid. The difference between the results for these two cases was applied as a correction to the value of $\bar{r}$ for the 100 × 100 case. The correction amounted to 20 per cent for the largest value of β and was negligible for small values of β.

If all the radii were set equal to zero at the beginning of the simulation, steady-state would never be reached because non-coalescing drops would stay in phase and would be detached in unison throughout the entire process. To offset this, the initial size distribution was set to be the distribution of Fig. 1. Steady state was normally reached in 5 iterations and 100 iterations were used to obtain an average value of $\bar{r}$. The averages and the standard deviations are shown in Fig. 5.

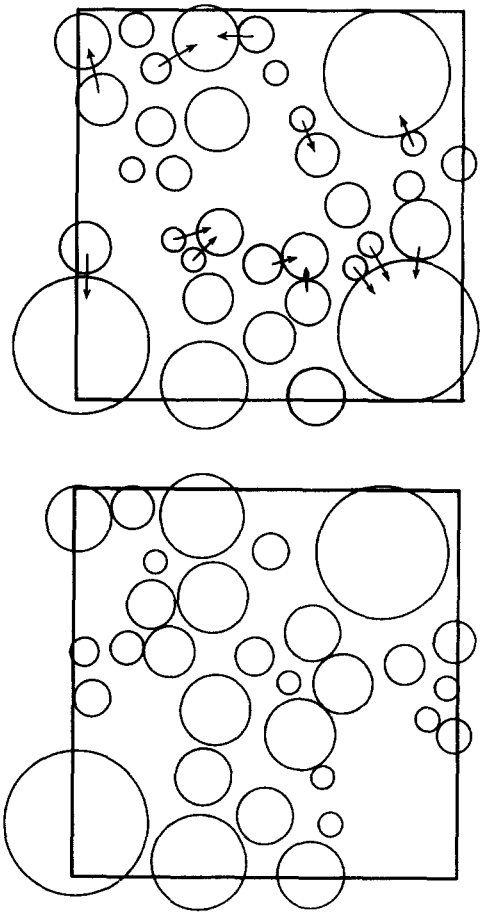

FIG. 3. Steady-state drop growth, coalescence, nucleation and drop removal in a typical computer sequence; $\beta = 7{\cdot}07$, $R = 18{\cdot}8$.

DISCUSSION

The dependence of $\bar{r}/R$ on the crowding factor β is shown in Fig. 5. From equations (5) and (6), the definition of h, and the fact that $a_2 = 2\pi/3$ for hemispherical drops, a relation

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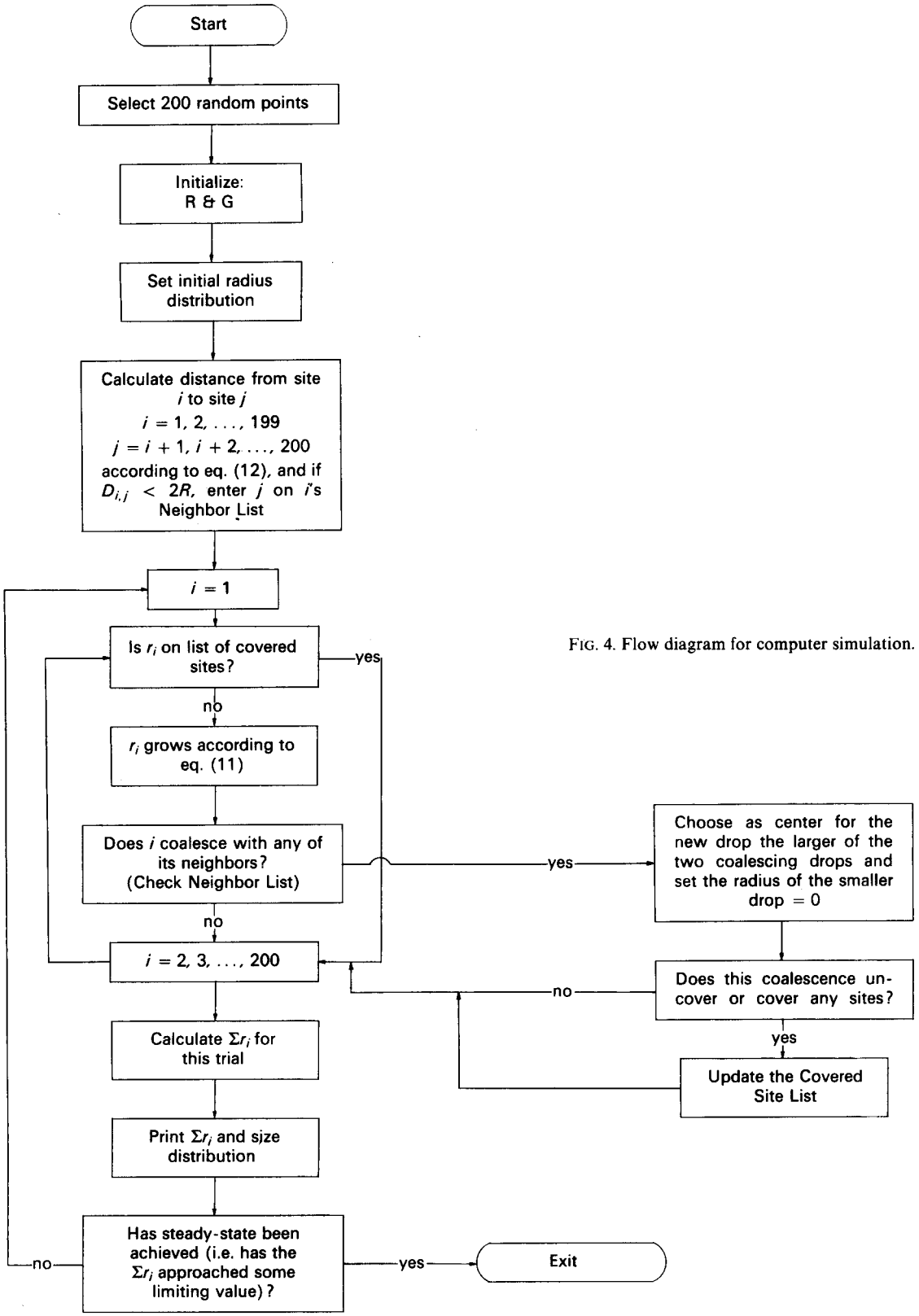

FIG. 4. Flow diagram for computer simulation.

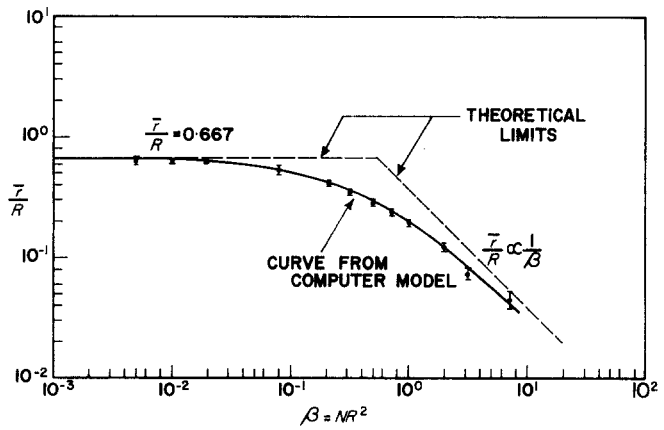

FIG. 5. Dependence of $\bar{r}/R$ on the crowding factor β for randomly distributed sites.

for the heat-transfer coefficient, h, can be derived

$$h = 13{\cdot}0k\bar{r}N. \tag{13}$$

The dependence of the heat-transfer coefficient on R and N can now be predicted using Fig. 5 and equation (13). In Fig. 6 the results of these calculations for water are shown. A value of $k = 1{\cdot}62 \times 10^{-3}$ cal/scm degC was assumed. For large nucleation site densities, the heat-transfer coefficient is dependent only upon the critical radius. For small critical radii (10^{-4} cm) and site densities greater than 10^8 sites/cm^2, the predicted heat-transfer coefficients are at least ten times greater than are actually observed in dropwise condensation on vertical condensers. This implies that extremely large heat-transfer coefficients could be obtained if a mechanism for efficient removal of these tiny drops from a condenser surface could be developed. It is interesting to compare experimental results for steady state condensation on vertical surfaces to the predictions of this model. In Fig. 6, the range of heat-transfer coefficients which have frequently been observed during drop condensation are shown. The model indicates that the nucleation site density must be greater than 5×10^4 sites/cm^2 to yield the heat-transfer rates obtained in conventional dropwise condensation experiments. Nucleation site densities slightly larger than this minimum value have been observed by Tammann and Boehme [11] and Peterson [12]; and have recently been estimated by McCormick and Baer [7] from the experimental measurements reported by Welch and Westwater [3]. In addition, the model indicates that the maximum drop radius before removal is less than 10^{-2} cm. Since drops on vertical surfaces are at least ten times larger than this before becoming unstable and sliding down the condenser surface, most of the drops must be removed by coalescing with other drops sliding down the surface, rather than by growing to the critical radius themselves. That is, drops grow to between 10^{-3} and 10^{-2} cm by vapor condensation and then chiefly grow to larger sizes by coalescence without any appreciable heat transfer.

CONCLUSIONS

1. A model for heat transfer during dropwise condensation on randomly distributed nucleation sites has been developed and simulated on a digital computer. The model accounted for drop growth, drop coalescence vacating active sites which existed beneath the smaller of the coalescing drops, re-nucleation on the newly

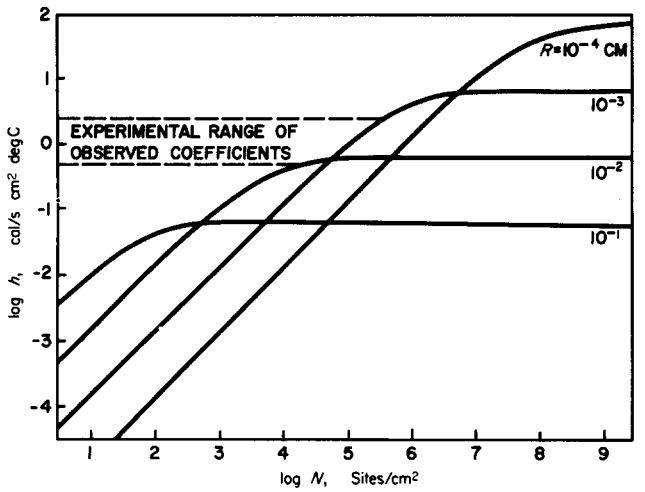

FIG. 6. Theoretical curves relating heat-transfer coefficient to nucleation site density and critical radius.

exposed sites, and drop removal. A typical case showing these events in a computer simulation is illustrated in Fig. 3.

2. The theory shows that in order to account for the heat-transfer coefficients in steady-state dropwise condensation, the minimum nuclei density is 5×10^4 sites/cm^2. This estimate is in good agreement with density values observed by several investigators. Also, it was shown that large drops, greater than 10^{-2} cm, grow chiefly by coalescence.

3. Heat-transfer coefficients many times larger than coefficients which have been experimentally observed to date are theoretically possible. To achieve these large coefficients, the condenser surface should contain at least 10^8 sites/cm^2 and small drops (10^{-4} cm) must be removed from the active site.

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Résumé—Un modèle pour le transport de chaleur au cours de la condensation par gouttelettes sur des sites de nucléation répartis au hasard a été exploité et simulé sur un calculateur numérique. La théorie tenait compte de la nucléation des gouttes et de la croissance, de la coalescence avec les gouttes voisines, de l'enlèvement et de la renucléation sur des sites exposés par les mécanismes d'enlèvement et de coalescence. Pour la condensation par gouttes en régime permanent, la théorie montrait que les petites gouttes croissent par condensation de la vapeur, et que les gouttes plus grandes croissent principalement par coalescence.

Des coefficients de transport de chaleur plus élevés que ceux observés jusqu'à présent seraient possibles si l'on pouvait mettre en oeuvre des surfaces de condensation contenant entre 10^7 et 10^8 sites par cm^2 et des techniques d'enlèvement de gouttes pour les très petites gouttes ($< 5 \times 10^{-4}$ cm).

Zusammenfassung—Eine Modellvorstellung der Wärmeübertragung durch Tropfenkondensation wurde entwickelt und auf einem Digitalrechner durchgerechnet. Eine statistische Verteilung der Keimzentren für entstehende Tropfen wurde angenommen. Die Theorie berücksichtigte das Entstehen und Wachsen von Tropfen, den Zusammenschluss benachbarter Tropfen, das Entfernen grosser Tropfen und das erneute Entstehen von Tropfen an Stellen, die durch Zusammenschlüsse und durch das Entfernen von Tropfen frei werden. Für stationäre Tropfenkondensation ergab sich aus der Theorie, dass die kleineren Tropfen durch Kondensation des Dampfes, die grösseren dagegen hauptsächlich durch Zusammenschlüsse wachsen. Grössere als bisher beobachtete Wärmeübergangszahlen müssten erreichbar sein, wenn die Kondensationsflächen 10^7–10^8 Keimzentren/cm^2 hätten und wenn eine Technik entwickelt werden könnte, mittels der man sehr kleine Tropfen ($< 5 \times 10^{-4}$ cm) entfernen kann.

Аннотация—Разработана модель теплопереноса при капельной конденсации на случайно расположенных центрах конденсации, которая затем была промоделирована на вычислительной машине. В анализе учитывалось зарождение и рост капель, столкновение с соседними каплями, удаление и образование новых капель на освободившихся центрах. Для стационарной капельной конденсации теоретически показано, что рост небольших капель происходит в результате конденсации пара, а более крупных — в основном в результате слияния. Коэффициенты теплообмена могут быть увеличены, если на поверхности конденсатора довести число центров образования капель до 10^7–10^8 на кв. см и разработать методику удаления мелких капель ($< 5 \times 10^{-4}$ см).